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Spatial 't Hooft loop, hot QCD and Z_N domain walls

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Abstract

We show that the deconfinement phase transition in the pure Yang-Mills theory can be characterized by the change of behaviour the spatial 't Hooft loop, $V(C)$. In the confining phase V has a perimeter law behaviour $V(C) \propto \exp\{-mP(C)\}$, while in the deconfined phase it has the area law behaviour $V(C) \propto \exp\{-\alpha S(C)\}$. We show that the area law behaviour of the 't Hooft loop is intimately related to the plasma-like distribution of the color charges in the hot QCD vacuum. We also show that the "dual string tension" α is equal to the"wall tension"of the Z_N domain walls previously calculated in [1]. All these properties generalize immediately to other nonabelian theories without fundamental charges, such as supersymmetric Yang Mills. In theories with fundamental charges the 't Hooft loop presumably has the area law behaviour already at zero temperature and therefore is not a good order parameter in the strict sense.

The deconfining phase transition in QCD is a subject with long history. It has seen a lot of activity in recent years primarily motivated by its possible relation with the heavy ion collision experiments at RHIC. Although the direct phenomenological relevance of the equilibrium high temperature QCD at RHIC is only conjectural, the subject of finite temperature behaviour of nonabelian theories is very fascinating theoretically and has possible cosmological applications in particular in relation to baryogenesis.

In this paper we will concentrate on a certain theoretical aspect of the hot phase in nonabelian gauge theories without fundamental matter fields. This class of theories is believed to have a deconfining phase transition to a "color plasma phase". Starting with early discussions of universality [2],[3] it has been studied quite extensively by both analytical perturbative [4] and lattice nonperturbative methods [5] and the nature of the phase transition is fairly well established.

Despite a lot of work on the subject which yielded very interesting results, it is perhaps surprising that a description of the phase transition in terms of a canonical order parameter does not exist. Although one frequently refers to the Polyakov line $P = \mathrm{Tr} P \exp\{ig \int dt A_0\}$ as an order parameter, it does not have the same status as the magnetisation in the Ising or Heisenberg model. While magnetisation is a canonical operator in the Hilbert space of the theory, the Polyakov line is not. It appears as an auxiliary quantity in the path integral formulation of the statistical sum, and while indeed it has qualitatively different behaviour in the confined and the deconfined phases, the "nonvanishing expectation value of P" above the phase transition can not be directly related to nonvanishing expectation value of any physical operator.

This ambiguity in the status of P several years ago has prompted discussions on the physical status of the "domain walls" - field configurations which interpolate between different "vacuum" values of P in the path integral [6],[7]. The outcome of this discussion has been itself somewhat unsatisfactory. On one hand as a result it is now understood that the "domain walls" are not physical objects of the same type as domain walls in the Ising model. They are not field configurations which interpolate between different physical states at spatial infinity. On the other hand the physical meaning of these walls has not been clarified. In particular it would be interesting to relate the "wall tension" (which has been calculated perturbatively [1] and nonperturbatively [8],[9]) to the expectation value of some physical observable.

The aim of this note is to rectify this situation. We will show that there indeed is a well defined order parameter whose behaviour changes qualitatively across the phase transition. This order parameter is the expectation value of the disorder variable - the spatial 't Hooft loop [10]. The operator $V(C)$ creates an elementary flux of magnetic field along a closed contour C. In the low temperature phase the VEV of V for large contours falls off according to a perimeter law, while in the high temperature phase it has an area law behaviour

$$< V(C) > \propto \exp\{-\alpha S(C)\} \tag{1}$$

The "dual string tension" α turns out to be precisely given by the "wall tension" of the Z_N domain wall. In this way the wall tension is directly related to a physical observable[1]. We hasten to add that the fact that the spatial 'tHooft loop has an area law does not mean that magnetic charges in the theory are confined by a linear potential. Just like the area law of the spatial Wilson loop does not indicate confinement of electric charges.

Let us start with recalling the definition of the disorder variable V [10]. In the following we will for definiteness consider the $SU(2)$ gauge group and will discuss the generalization to other groups later. The defining property of V is that it satisfies the following commutation relation with the spatial fundamental Wilson loop W

$$V^{\dagger}(C)W(C')V(C) = e^{i\pi n(C,C')}W(C') \tag{2}$$

where $n(C, C')$ is the linking number of the curves C and C'.

The operatorial representation of V is constructed as follows. Consider a closed contour C which lies in the xy plane. Define the function $\omega_C(x)$ which is equal to the solid angle subtended by C as seen from the point x. The function ω is continuos everywhere, except on a surface S bounded by C, where it jumps by 4π. Other than the fact that S is bounded by C, its location is arbitrary. Now consider the operator of the "singular gauge transformation" in the third colour direction with the gauge function $\omega/2g$

$$V(C) = \exp\{\frac{i}{2g}\int d^3x(\bar{D}^i_{3a})\omega_C E^i_a\} \tag{3}$$

The bar over the covariant derivative indicates the fact that only the regular part of the derivative of ω enters in the definition of the operator V. An alternative, and a somewhat simpler form of V is obtained by using the fact that on the physical states $D^iE^i = 0$ and that the solid angle $\omega_C(x)$ vanishes at infinity. Therefore on physical states

$$V(C) = \exp\{\frac{2\pi i}{g}\int_S d^2S^iE^i_3\} \tag{4}$$

The integration here is over the surface S on which ω is defined to have the discontinuity. If no fields in the fundamental representation are present in the theory, the operator V does not depend on this surface, but rather depends only on its boundary C. To see this, note that changing S to S' adds to the phase $\frac{2\pi}{g}\oint_{S-S'} d^2S^iE^i$. In a theory with only adjoint charges the charge within any closed volume is a multiple integer of the gauge coupling $\oint_{S-S'} d^2S^iE^i = gn$ and therefore the extra phase factor is always unity. In the following we will for simplicity always choose the surface S to lie in the xy plane.

[1] We note that the 2+1 dimensional analog of the 't Hooft loop was discussed in the context of the finite temperature gauge theory in [9]. However the change of its behaviour across the phase transition was not analysed in that work.

It is easy to see that

$$V^\dagger(C) A_i^a(\mathbf{x}) V(C) = A_i^a(\mathbf{x}) + a_i^a(\mathbf{x}) \tag{5}$$

with

$$a_i^a(\mathbf{x}) = \delta^{a3}\delta_{i3}\frac{2\pi}{g}\delta(\mathbf{x} - S) \tag{6}$$

The δ-function in this equation is one dimensional and is defined such that its integral along a curve normal to the surface S is equal to unity.

The operator $V(C)$ is therefore seen to create an infinitely thin elementary vortex of magnetic field in the third color direction along the curve C. Clearly our choice of the third direction in the color space is arbitrary. One can equaly well consider any other direction. Different operators defined in this way transform into each other under the gauge transformations and therefore are identical when acting on physical states.

The operator V was introduced by 't Hooft in order to study the phase structure of gauge theories. In the zero temperature ground state of the pure Yang-Mills theory the Wilson loop has an area law behaviour. 't Hooft's general analysis showed that (unless the gauge symmetry was partially broken) the area law behaviour of 't Hooft loop and Wilson loop were mutually exclusive. The 't Hooft loop therefore has a perimeter law behaviour in the ground state. This statement however pertains only to the zero temperature ground state, which is Lorentz invariant and where the spacelike and timelike loops behave in the same way. Our purpose here is to study the expectation value of the 't Hooft loop at high temperature.

The expectation value of the 't Hooft loop at finite temperature is given by the following expression

$$< V(C) >= \mathrm{Tr} e^{-\frac{\beta}{2}(E^2+B^2)} e^{i\frac{2\pi i}{g}\int d^2S^i E_3^i} \tag{7}$$

Note that formally eq.(7) is the same as for the partition function except that the Hamiltonian has an extra term linear in the electric field

$$\delta H = -iT \int d^3x a_a^i E_a^i \tag{8}$$

with a_a^i defined in eq.(6) and $T = \frac{1}{\beta}$. With the help of the standard manipulations it is easy to cast this expression in the path integral form. Introducing the imaginary time axis and the Lagrange multiplier field A_0 in the standard way we obtain

$$< V >= \int DA_i DA_0 \exp\{-\frac{1}{2}\int_0^\beta dt \int d^3x\, (\partial_0 A_i^a - (D_i A_0)^a - T a_i^a)^2 + (B^a)^2\} \tag{9}$$

Our task is now simple. In this expression the "external field" a_i enters only in one place - it shifts the spatial derivative of the zero Matsubara frequency mode

of A_0. The integration of the nonzero Matsubara modes therefore proceeds in precisely the same way as in the standard calculation of the finite temperature effective potential [11],[12]. In fact we can take the whole page from the book of [12] and derive the constrained effective Lagrangian for the zero Matsubara frequency modes in the presence of the external field a_i. It is obvious that the calculation is identical to that of [12]. For simplicity we will restrict ourselves to the one loop result

$$S_{eff} = \frac{2T^2}{g^2}(\partial_i q + \frac{g}{2}a_i)^2 + \frac{4}{3}T^4 q^2(1-\frac{q}{\pi})^2 \tag{10}$$

Here q is defined ([12]) as the average value of the first eigenvalue of the matrix $A_0 = \frac{A_0^a\tau^a}{gT}$ at zero Matsubara frequency. The matrices τ^a are the generators of $SU(2)$ in the fundamental representation and are normalized according to $\mathrm{tr}\tau^a\tau^b = \frac{1}{2}\delta^{ab}$.

To find the average of V we have to find the configuration of q which minimizes the action eq.(10). Qualitatively the form of the solution is clear. Let us consider a large 't Hooft loop such that its radius is much larger than the electric mass in eq.(10). Clearly very far from the loop at spatial infinity the field q must take the value which minimizes the potential term in eq.(10). There are two such values $q = 0, \pi$. For definiteness we take the asymptotic value at infinity to be 0. On the other hand on the surface S, where the external field a_i is a delta function, q has to jump by π. It is also clear that for a large loop the solution must be practically x and y independent as long as x and y are well within the surface S. For these values of x and y therefore we are looking for the z - dependent solution which everywhere except at $z = 0$ is a solution of free equations (the source vanishes), while at $z = 0$ jumps by π and is constrained to approach 0 at $z \to \pm\infty$. But of course we know what this solution is. Recall that the action eq.(10) without the source term allows wall-like solutions q_{wall}. They interpolate between $q = 0$ at $z \to -\infty$ and $q = \pi$ at $z \to +\infty$. This is precisely the Z_2 wall alluded to earlier. Let us pick the profile which corresponds to the wall at $z = 0$. This means that $q_{wall}(0) = \frac{\pi}{2}$. We can now construct the following function

$$q_S(z) = \begin{array}{lll} q_{wall}(z) & , & z < 0 \\ q_{wall}(z) & -\pi, & z > 0 \end{array} \tag{11}$$

This function has precisely the required properties: it satisfies the correct boundary conditions, has the correct discontinuity and solves the sourceless equations everywhere except at discontinuity. Close to the boundary of S the solution will be of course modified - it is not x, y independent anymore. We will not address here the question of what the exact form of the solution is[2]. It is however clear that the behaviour of q close to the boundary of S does not affect the leading

[2] We only note that disregarding the boundary would give one infinity of wall solutions shifted with respect to each other in the z direction. The main effect of the boundary terms is to fix the wall centered on the plain containing the contour C.

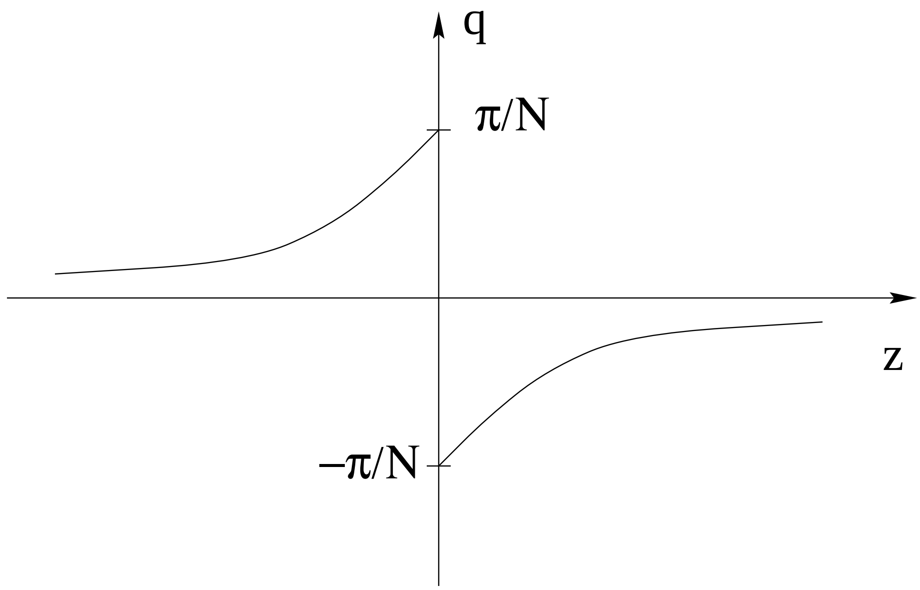

Figure 1: The profile of the field A_0 that dominates the steepest descent calcu-laltion of V.

contribution to the action. This leading contribution for the large 't Hooft loop is obviously proportional to the minimal area S_m subtended by the contour C

$$S_{eff} = \alpha S_m \tag{12}$$

where α is precisely the wall tension of the Z_2 wall.

Note that the action is proportional to the minimal area S_m subtended by the contour C rather than the area of the surface S which appears explicitly in the definition of the operator V and eq.(6). Independence on S is of course a consequence of the fact mentioned earlier, that the operator V is independent of S in the absence of fundamental charges. In the framework of our path integral calculation this is easy to understand. Let us consider the operator V defined on some other surface defined by equation $z = f(x, y)$ rather than $z = 0$, but which has the same boundary[3]. This means that the source term in eq.(10) is modified accordingly and induces the jump by π on this new surface. The solution to the equations of motion with the required singularity structure now is

$$\begin{aligned} q_S(z) = \quad & q_{wall}(z) \quad , \qquad z < f(x, y) \\ & q_{wall}(z) \quad -\pi, \quad z > f(x, y) \end{aligned} \tag{13}$$

[3]For simplicity, here and in the rest of this note we consider only surfaces which are smooth on the scale of the electric screening length.

Since the potential term in eq.(10) is symmetric under shift of q by π, the action of this solution is obviously the same as of eq.(12) and is proportional to the minimal area S_m.

We therefore conclude that the expectation value of a large 't Hooft loop follows an area law

$$< V(C) >= e^{-S_{eff}} = e^{-\alpha S} \tag{14}$$

The generalization of the preceding discussion to $SU(N)$ gauge theories is straightforward. The object of interest here is the 't Hooft loop

$$V(C) = \exp\{\frac{i}{gN}\int d^3x Tr(\bar{D}^i\Omega_C)E^i\} \tag{15}$$

with $\Omega_C(x)$, as before, the singular (solid angle) gauge function in the hypercharge direction

$$\Omega_C(x) = \omega_C(x)Y \tag{16}$$

where the hypercharge generator Y is defined as

$$Y = \mathrm{diag}\,(1, 1, ..., -(N-1)) \tag{17}$$

The calculation of $< V(C) >$ proceeds exactly along the same lines as before. In the path integral representation[4]

$$< V >= \int DA_i DA_0 \exp\left\{-\int_0^\beta dt\int d^3x Tr\left[\left(\partial_0\vec{A} - \vec{D}A_0 - T\vec{a}\right)^2 + (\vec{B})^2\right]\right\} \tag{18}$$

with

$$a^i(\mathbf{x}) = \frac{2\pi}{gN}Y\delta^{i3}\delta(\mathbf{x} - S) \tag{19}$$

The effective action is easily calculated again following [12]. The result is simple for the configurations of the vector potential of the form

$$A_0 = \frac{qT}{g}Y \tag{20}$$

The one loop result is

$$S_{eff} = \frac{T^2}{g^2}N(N-1)\left(\partial_i q(x) - \frac{2\pi}{N}\delta^{i3}\delta(x - S)\right)^2 + \frac{N^2(N-1)}{3}T^4q^2\left(1 - \frac{Nq}{2\pi}\right)^2 \tag{21}$$

One again obtains the area law for $< V(C) >$ with the dual string tension given by the wall tension of the Z_N "domain wall".

[4]We have switched to the standard matrix notation where $E = E^a\lambda^a$ with the fundamental representation $SU(N)$ generators normalized according to $Tr\lambda^a\lambda^b = \frac{1}{2}\delta^{ab}$.

The following comment is in order here. As we mentioned earlier 't Hooft gave an argument to the effect that in a Lorentz invariant state the Wilson loop and the 't Hooft loop can not have simultaneously an area law behaviour. As is well established, the spatial Wilson loop at high temperature in fact does have an area law. Nevertheless this fact is not in contradiction with our result. The point is that at nonzero temperature the symmetry between a spatial and a temporal loop is broken. 't Hoofts argument [10] then only rules out the simultaneous area law behaviour for the spatial 't Hooft loop and a timelike Wilson loop. In the plasma phase in fact the string tension for timelike Wilson loops disappears, and our result is therefore only natural.

Strictly speaking our proof applies at very high temperatures where the perturbative calculation of $< V(C) >$ is valid. However keeping in mind that at zero temperature V has a perimeter law behaviour the most natural possibility is that the change in the behaviour occurs at the deconfining phase transition. In fact the same argument by 't Hooft already mentioned before also claims that either the 't Hooft loop or its dual must have an area law if no massless particles are present in the theory. Since one does not expect massless excitations to exist at any temperature, this line of reasoning would tell us that the dual string tension has to appear exactly at the same temperature at which the timelike Wilson loop ceases to confine, i.e. at the phase transition.

To strengthen this argument we would like to present a simple discussion of why the area law of the 't Hooft loop is intimately related to the plasma like distribution of charges in the equilibrium state.

Consider an equilibrium neutral plasma of electric charges with the statistical sum

$$Z = \sum_{\{\rho\}} e^{-\beta \sum_{x,y} e^2 \rho(x) D(x-y) \rho(y)} \tag{22}$$

Here ρ is the integer valued charge density

$$\rho(x) = \sum_{x_i} n_i \delta(x - x_i) \tag{23}$$

The integer n_i count how many positive or negative charges reside at the point x_i. The interaction between the charges is via the Coulomb potential $D(x-y)$

$$D(x-y) = - < x|\frac{1}{\partial^2}|y > \tag{24}$$

The physics of this system is well known. Dynamically the screening length is generated and the "dressed" potential between the charges is screened on the distance scale ξ. For dilute plasma the screening length is inversely proportional to the fugacity μ. The convenient standard way to study this system is by duality transformation [13]:

$$Z = \int D\phi \sum_{\{\rho\}} e^{-\frac{T}{4e^2}\partial\phi\partial\phi + i\rho\phi} \tag{25}$$

The scalar field ϕ "classically" is related to the charge density by

$$\partial_i^2\phi = -i2e^2\beta\rho \tag{26}$$

Summing over ρ this partition function is rewritten as

$$Z = \int D\phi e^{-\frac{T}{4e^2}\int(\partial_i\phi)^2+V(\phi)} \tag{27}$$

In the dilute plasma limit the potential

$$V = \mu^2(1-\cos\phi) \tag{28}$$

When the plasma is not dilute the explicit form of V is not easy to calculate but it is still true that V is necessarily a periodic function of ϕ with period 2π. This periodicity is of course a direct consequence of the the integer valuedness of the charge density eq.(23) and the way the field ϕ was introduced in eq.(25) Therefore V has discrete minima and the classical equations for ϕ have wall like solutions. Now let us calculate the average of the 'tHooft loop. As we have discussed earlier, the 't Hooft loop is a simple operator which measures the electric flux through the area bounded by a contour C. The electric field is given in terms of the charge density as

$$\partial_i E_i = e\rho \tag{29}$$

A short calculation shows that the electric flux through a contour C due to a point charge at the point x is proportional to the solid angle ω. Therefore with the appropriate normalization the 't Hooft loop operator is

$$V(C) = e^{\frac{i}{2}\int d^3x\rho(x)\omega_C(x)} \tag{30}$$

It is now obvious that the calculation of the 't Hooft loop in this system is *verbatim* equivalent to Polyakov's calculation of the Wilson loop in the plasma of magnetic monopoles [14]. The loop therefore has the area law with the dual string tension equal to the wall tension of the classical wall solution of eq.(27).

This simple discussion is a very close caricature to the QCD calculation presented in this note. Recall, that the electric field in QCD in the Euclidean finite temperature calculation is related to the scalar potential by

$$E^i \propto i\partial_i A_0 \tag{31}$$

This together with the Gauss' law

$$\partial_i E_i = \rho \equiv gA_i \times E_i \tag{32}$$

shows that A_0 is the exact counterpart of ϕ if one thinks of the charges that constitute the plasma as the colour charged gluons. More precisely, the QCD analog of the charge in the simple plasma calculation is the hypercharge Y. Of

course, the QCD plasma at high temperature is not dilute, and as a result the potential V is not quite the simple sine-Gordon potential as in [14]. It does however preserve the basic feature of periodicity. The basic physics does not depend on the exact shape of the potential and therefore as far as the 't Hooft loop is concerned it is the same at high temperature QCD and in the simple equilibrium plasma of charges.

What happens in the confining phase? Of course the physics there is nonperturbative and therefore no quantitative statements are available. It is however easy to understand the basic features. Since there are no free charges, the fugacity in the confining phase vanishes. Consequently there is no potential that suppresses fluctuations of A_0 (sic. ϕ) - no Debye mass, and the dual string tension vanishes.

We note peripherically to our main discussion, that the plasma picture gives a natural explanation of a somewhat unusual scaling of the domain wall tension with N. The area law of the 't Hooft loop is due to the presence in the plasma of gluons with nonzero hypercharge Y. There are $N-1$ gluons with $Y=N$ and $N-1$ gluons with $Y=-N$, while the rest of the $(N-1)^2$ are hypercharge neutral. A gluon with the hypercharge $\pm N$ which sits close to the minimal area spanned by the contour C contributes a factor -1 to $<V>$. This is due to the fact that only half of the electric flux emanating from this gluon, crosses the area of the loop. For k gluons the factor is obviously $(-1)^k$. Due to the plasma screening effect, only those gluons that are at a distance smaller than the screening length $1/m$ are effective in the disordering of the dual loop. Let us for simplicity assume that the distribution of the number of gluons in the plasma at distances scales smaller than the correlation length $1/m$ is random and follows the Poisson distribution.

$$P(k)=\frac{\bar{k}^k}{k!}e^{-\bar{k}} \tag{33}$$

Here $\bar{k}$ is the average number of gluons in the disk of thickness $1/m$ around the minimal area S spanned by the loop. The average of the loop is then estimated as

$$<V>=\sum_k P(k)(-1)^k=e^{-2\bar{k}} \tag{34}$$

Now

$$\bar{k}=\frac{1}{m}S_m n(T) \tag{35}$$

where $n(T)$ is the density of the charged gluons in the plasma. Since there are $2(N-1)$ species of charged gluons the density in the large N limit scales as

$$n\propto N \tag{36}$$

The screening length $1/m$ at large N is finite, $m^2\propto g^2N$. The dual string/wall tension therefore scales as

$$\alpha=\frac{1}{m}n\propto\frac{N}{\sqrt{g^2N}} \tag{37}$$

This is precisely the scaling found in the semiclassical calculation [1].

The last point we want to discuss is the fate of the 't Hooft loop in a theory which contains fundamental charges. Here we expect V to have an area law already at zero temperature which presumably then survives at arbitrary temperature. In fact the situation is slightly more intricate. Recall that our definition of V eq.(3) did not depend on the surface S only in the absence of fundamental charges. In the presence of such charges, the operator V depends explicitly not only on the contour C but also on the surface S on which the angular function ω is defined to have the discontinuity. In fact it creates a current across the surface S [15]. As a result one expects the average of V will have an area law of the form

$$V_S(C) = \exp\{-\tilde{\alpha} S\} \tag{38}$$

where S is not the minimal area subtended by C, but rather the area of the surface which enters explicitly in the definition of V. The dual string tension $\tilde{\alpha}$ is due to the vacuum fluctuations of the fundamental charges and should therefore be a decreasing function of the mass of the lightest fundamentally charged field.

At high temperature this expectation is easily confirmed by a simple analysis. Consider as an example the theory with n_f flavours of fundamental fermions. Repeating our calculation we obtain the analog of eq.(10) but with potential which is not periodic in q and has therefore only one minimum at $q = 0$[12].

$$\begin{aligned} V_{eff} &= \frac{4}{3}\pi^2 T^4 N^2 (N-1)\left(\frac{q}{2\pi}\right)^2 \left(1 - \frac{Nq}{2\pi}\right)^2 \\ &- \frac{4}{3}\pi^2 T^4 n_f \left[(N-1)\left(\frac{q}{2\pi}+\frac{1}{2}\right)^2\left(\frac{q}{2\pi}-\frac{1}{2}\right)^2 + \left(\frac{(N-1)q}{2\pi}+\frac{1}{2}\right)^2\left(\frac{(N-1)q}{2\pi}-\frac{1}{2}\right)^2\right] \end{aligned} \tag{39}$$

The minimum at $q = \pi$ has now higher energy and therefore is only metastable. Such a potential of course does not have a stable wall solution. Let us now consider the 't Hooft loop operator which is defined on a surface $z = f(x, y)$. For simplicity let us assume that for x and y inside the contour C, the function f is large and negative. Then the solution of the equations of motion we are looking for will be very close to the vacuum value $q = 0$ just to the left of the surface S. However, since the solution is forced to have a discontinuity across S, the field will be equal to π just to the right of S. Since the potential is not degenerate at 0 and π, the region adjacent to the wall form the right has nonvanishing potential energy. Clearly in the minimal action solution the field will tend to the vacuum value $q = 0$ within a layer of thickness of order of the Debye mass "glued" to the surface S. The action of such a configuration is proportional $S_{eff} \propto \tilde{\alpha} S$.

At low temperatures the semiclassical analysis is not adequate. However the fluctuations of the fundamental charges are still present in the vacuum. Since the 't Hooft loop creates the fundamental current through the surface S it is clear that the vacuum in the presence of the loop is modified everywhere

along S, and therefore the 't Hooft loop must have the same type of area law behaviour as in eq.(38). We therefore conclude that, just like the Polyakov line, the 't Hooft loop ceases to be an order parameter in the strict sense: it has the area law behaviour on both sides of the deconfining phase transition. We expect however, that again just like with the Polyakov line, even though in the dual string tension α is nonvanishing everywhere, it jumps strongly across the phase transition and therefore in practical sense should be a good indicator of the transition. The intuitive basis for this expectation in QCD is the chiral symmetry restoration in the plasma phase. Below the phase transition the quarks have a dynamically generated mass, which disappears above the critical temperature. The dual string tension $\tilde{\alpha}$ must have inverse dependence on the quark mass, since it vanishes in the confining phase for infinitely heavy quarks. Therefore one can expect a jump in $\tilde{\alpha}$ at the phase transition which is of order $\tilde{\alpha}$ itself.

Finally we note that the loop average can be measured on the lattice. The lattice version of V is [16]:

$$V = \exp \beta_L (1 - \exp i2\pi/N) \sum_{x \in S} \left(\frac{1}{N} Tr P_{zt}(x) + c.c \right) \tag{40}$$

where β_L the lattice coupling and P_{zt} are the electric plaquettes orthogonal to the surface. It was precisely this operator, with the surface S covering *all* of a given x-y cross section in a periodic volume, that was used by Kajantie et al.[1]. Thus the dual string tension of the 't Hooft loop and the wall tension measured by this group must be identical. It would be desirable to have results closer to the continuum limit.

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